Determination of Full Elastic Constants of Carbon Fiber in Carbon Fiber Reinforced Plastic Composites

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ABSTRACT

The full elastic constants of a carbon fiber in a unidirectional carbon fiber reinforced plastic composite were investigated by using a resonant ultrasound spectroscopy technique. The equivalent elastic properties of the composite were determined by comparing resonance frequency obtained from experiment and analysis. The subspace method which is the kind of the iterative method for calculating eigenvalues and eigenvectors of the real symmetric matrix such as the stiffness and mass matrix, was used to estimate the resonance frequency of the composite. The error function was calculated using the natural frequency obtained by the experiment and finite element analysis. It was found that the estimated elastic constants of the carbon fiber in the composite based on both Eshelby-Mori-Tanaka theory were reasonably consistent with the previously-reported values. The effectiveness of the method used here was confirmed by comparing the experimental results and those obtained by numerical analysis for the carbon fiber in the composite. The technique could be applicable to any other continuous fibers including those with much smaller diameter, as well as randomly oriented discontinuous fibers.

INTRODUCTON

Composite materials are attracting the attention of people with in engineering sectors because of the unique mechanical characteristics and ease of property customization, thereby making them highly competitive with conventional materials. Carbon fiber reinforced plastics (CFRPs), which are a common class of composite material, are increasingly being used as lightweight and high-stiffness materials in various applications. The process of determining the potential amount of weight that can be saved requires that the fracture properties of the CFRP in the direction of the fiber axis be a major consideration in the design of composite structures. Thus, accurate prediction in the mechanical properties of the unidirectional CFRP composite continues to be central to CFRP composite research. However, measurements of the

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full elastic constants of carbon fibers is acknowledged to be difficult due to the small size of these fibers.

According to previous reports, Miyagawa et al. [1] investigated transverse elastic modulus values of carbon fibers by using tensile-loading and nanoindentation technique. The strain of carbon fibers under tensile loading was measured using the laser in Raman spectroscopy. The transverse modulus of the carbon fiber was calculated from the load-displacement curves according to the procedure reported by Oliver and Pharr [2]. The theoretical transverse modulus of carbon fibers calculated using the Mori-Tanaka [3], Halpin-Tsai [4], and Uemura [5] equations. Tanaka et al. [6] reported the effect of nanostructure upon the deformation micromechanics of carbon fibers. In order to determine the elastic constants of the carbon fibers, the micromechanical model taking into account both the crystallites and amorphous components of the fiber structure was implemented. The elastic coefficients of carbon fiber were calculated using Eshelby solution [7] and Mori-Tanaka's mean stress method [3] from the elastic coefficients of the amorphous and crystallite components in the fibers. Csanadi et al. [8] reported the three-phase Eshelby-Mori-Tanaka micromechanical models which takes into account both crystalline, amorphous phases and microvoids of the fiber structure.

In this study, the identification of the full elastic constants of the carbon fiber in the unidirectional CFRP composite by using the RUS method and multiscale analysis was conducted. The RUS method is the kind of a non-destructive testing method, and able to identify the full elastic constants of elastic materials. The estimation of the natural frequency of the unidirectional CFRP composite using finite element analysis was performed. Then, the natural frequency of the composite was measured experimentally, and the optimization using the natural frequency obtained from the experiment was performed in order to identify the full elastic constants of the composite. Furthermore, the multiscale analysis was performed in order to estimate the full elastic constants of the carbon fiber in the unidirectional CFRP composite.

EXPERIMENTAL AND ANALYSIS METHOD

High-strength and high-modulus polyacrylonitrile (PAN)-based carbon fiber (T700S, TORAY) and a bisphenol-A epoxy resin material were used to prepare the unidirectional CFRP composite. The composite was prepared via a conventional hot-pressing technique so as to produce the laminate structure of $[0_{16}]$. The fiber volume fraction and bulk density of the resultant composite were 54% and 1.57 Mg/m^3, respectively. Then the composite was cut into 4.069 mm (width) × 3.090 mm (length) × 2.038 mm (thickness) using a wheel saw.

The composite was fixed between two coaxially aligned acoustic emission (AE) sensors (F50a sensors, Physical Acoustics). One transducer excites the vibration of the composite, and the other monitors the response and detects resonance frequencies. By inputting the sine wave signal at 20 V using the function generator (WF1974, NF Corporation), the composite was vibrated, and the vibration generated on the composite was detected by the other sensor. The detected signal was amplified using the preamplifier (0/2/4, Physical Acoustics), and the frequency response of the composite was measured using the spectrum analyzer (MOD3014, Tektronix). The natural frequency of the composite was measured from 100 kHz to 500 kHz in 0.1 kHz steps.

The elastic constants of the unidirectional CFRP composite were estimated from the assumed elastic constants of the carbon fiber using the Mori-Tanaka equivalent inclusion method. The elastic constants of the epoxy material and carbon fiber [9] used for the estimation of composite elastic modulus are summarized in TABLE I and TABLE II, respectively. Here, the subscripts L and T denote the longitudinal (fiber axis direction) and transverse directions, respectively. The elastic constants of the composite $\boldsymbol{D}$ was estimated using M-T method. The elastic stiffness matrix used for calculation can be given as following equation.

$$\boldsymbol{D} = \boldsymbol{D}_{\mathrm{m}} \left\{ (1-f)(\boldsymbol{D}_{\mathrm{f}} - \boldsymbol{D}_{\mathrm{m}}) S + \boldsymbol{D}_{\mathrm{m}} \right\}^{-1} \left[(1-f)\left\{ (\boldsymbol{D}_{\mathrm{f}} - \boldsymbol{D}_{\mathrm{m}}) \boldsymbol{S} + \boldsymbol{D}_{\mathrm{m}} \right\} + f\boldsymbol{D}_{\mathrm{f}} \right], \tag{1}$$

where $\boldsymbol{D}_{\mathrm{f}}$, $\boldsymbol{D}_{\mathrm{m}}$ and $\boldsymbol{S}$ are the elastic stiffness matrix of the carbon fiber, epoxy material, and Eshelby's matrix. The f is the volume fraction of the carbon fiber in the composite. The elastic constants of the composite are summarized TABLE III.

The natural frequency was estimated from the elastic constants of the unidirectional CFRP composite using the finite element analysis. The mode shape and natural frequency values obtained by finite element analysis is shown in Figure 1 and TABLE □, respectively.

TABLE I. MATERIAL PROPERTIES FOR THE EXPOXY MATERISL

Young's modulus, E	3.4 GPa
Poisson's ratio, ν	0.31

TABLE II. MATERIAL PROPERTIES FOR THE CARBON FIBER [9]

Longitudinal Young's modulus, E_{LL}	243.0 GPa
Transverse Young's modulus, E_{TT} and E_{TT}	13.8 GPa
Shear modulus, G_{12} and G_{13}	23.1 GPa
Shear modulus, G_{23}	5.0 GPa
Poisson's ratio, ν_{12} and ν_{13}	0.29
Volume fraction, f	0.54

TABLE III. ELASTIC CONSTANTS OF THE UNIDIRECTIONAL CFRP COMPOSITE

Longitudinal Young's modulus, E_{LL}	132.8 GPa
Transverse Young's modulus, E_{TT} and E_{TT}	6.7 GPa
Shear modulus, G_{12} and G_{13}	3.7 GPa
Shear modulus, G_{23}	2.4 GPa
Poisson's ratio, ν_{12} and ν_{13}	0.30

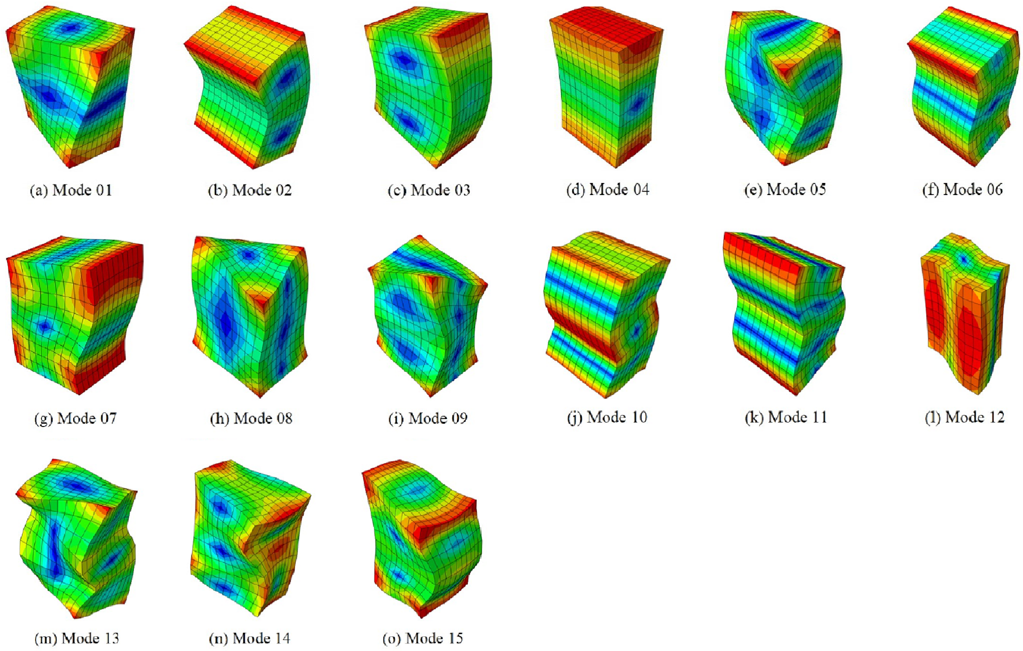

Figure 1. Mode shape for resonance frequencies of the unidirectional CFRP composite.

TABLE IV. RESONANCE FREQUENCY OF THE COMPOSITE ESTIMATED BY FE ANALYSIS

Mode number	Frequency (kHz)	Mode number	Frequency (kHz)
Mode 1	143.874	Mode 10	386.141
Mode 2	164.161	Mode 11	391.141
Mode 3	197.690	Mode 12	403.184
Mode 4	248.578	Mode 13	404.987
Mode 5	266.867	Mode 14	406.926
Mode 6	268.512	Mode 15	406.969
Mode 7	268.907	Mode 16	427.090
Mode 8	302.038		
Mode 9	352.262		

The elastic constants of the unidirectional CFRP composite and carbon fiber were determined by comparing the natural frequency obtained from the experiment and analysis. The elastic constants of the carbon fiber in the unidirectional CFRP composite was set, and then the stiffness matrix and the mass matrix of the composite were calculated using finite element method. The eigenvectors, eigenvalues and eigenfreqencies were calculated from the stiffness matrix and the mass matrix using the subspace method and the generalized Jacobi method. Next, the error function was calculated using the natural frequency obtained by the experiment and the analysis; this is given as

$$F = \sum_i w_i \left(1 - \frac{\overline{f_i}}{f_i}\right)^2, \qquad (2)$$

where $\overline{f_i}$, f_i, and w_i are the *i*-th vibration mode of the natural frequency obtained by the analysis and experiment, respectively, and *i*-th weighting factor. The purpose of the error function is to reduce the difference between the natural frequency obtained from the experiment and the natural frequency obtained from the analysis. If the *i*-th vibration mode of the natural frequency obtained by the experiment, the i-th weighting factor is "one". If the *i*-th vibration mode of the natural frequency did not obtain by the experiment, the *i*-th weighting factor is "zero".

RESULTS AND DISCUSSION

The response spectrum of the unidirectional CFRP composite is shown in Figure 2; the several peaks can be observed from 100 kHz to 500 kHz. The natural frequency obtained by the experiment and analysis are summarized in TABLE . The natural frequency obtained by the analysis was close to the natural frequency obtained by the experiment. The elastic constants of the carbon fiber calculated via M-T method were shown in TABLE . Different types of PAN-based carbon fibers (T650 and T300) are also indicated. It was found that the estimated elastic constants of the carbon fiber in the unidirectional CFRP composite based on both Eshelby-Mori- Tanaka theory were reasonably consistent with the previously-reported values. The effectiveness of the method used here was confirmed by comparing the experimental results and those obtained by numerical analysis for the carbon fiber in the unidirectional CFRP composite. The technique presented here could be applicable to any other continuous fibers including those with much smaller diameter, as well as randomly-oriented short fibers.

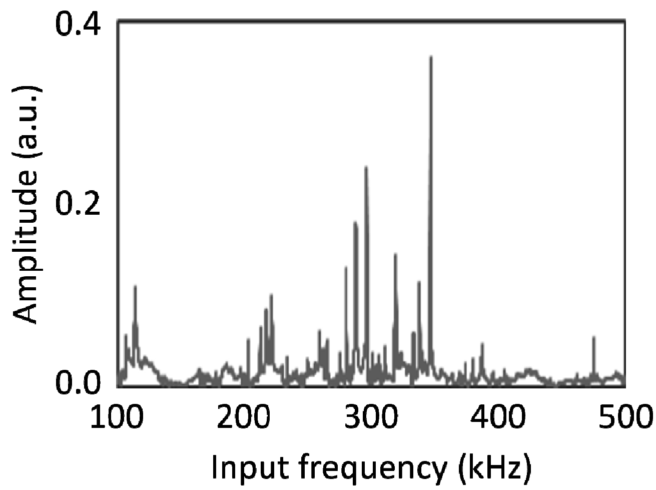

Figure. 2. Response spectrum of the unidirectional CFRP composite.

TABLE □. RESONANCE FREQUENCY OF THE UNIDIRECTIONAL CFRP COMPOSITE

Mode number	Experiment (kHz)	Analysis (kHz)
Mode 3	202.55	197.690
Mode 4	249.10	248.578
Mode 7	279.90	268.907
Mode 8	321.45	302.038
Mode 9	355.35	352.262
Mode 16	437.80	427.090

TABLE □. ESTIMATED ELASTIC CONSTANTS OF THE CARBON FIBERS. SHOWN ARE THE LONGITUDINAL AND TRANSVERSE YOUNG'S MODULUS (E_{LL} AND E_{TT}), THE IN-PLANE AND OUT-OF-PLANE POISSON'S RATIOS (ν_{LT} AND ν_{TT}), AND THE OUT-OF- PLANE SHARE MODULUS (G_{LL} AND G_{TT}).

Fiber types	E_{LL}	E_{TT}	ν_{LT}	ν_{TT}	G_{LT}	G_{TT}	Source
T700S	239.4	13.0	0.47	0.17	45.4	5.6	This study
T700S	230.0	-	-	-	-	-	Ref [10]
T700S	177.9	26.1	0.27	0.77	23.9	7.4	Ref [11]
T650	243.0	13.8	0.29	0.38	23.1	5.0	Ref [9]
T300	204.5	14.6	0.27	0.47	22.8	5.0	Ref [9]

CONCLUSION

In this study, the full elastic constants of the carbon fiber in the unidirectional CFRP composite were identified. The composite was prepared via hot-pressing method. The natural frequency of the composite was measured, and the full elastic constants of the carbon fiber were identified using Mori-Tanaka equivalent inclusion method and finite element method. It was found that the full elastic constants of the carbon fiber in the unidirectional CFRP composite could be obtained approximately by M-T method. We have also demonstrated that the measured elastic constants of the carbon fiber in the unidirectional CFRP composite were reasonably consistent with the previous reported values.

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